
Inventory Management: Economic Order Quantity, JIT, and the Theory of Constraints


## AFTER STUDYING THIS CHAPTER, YOU SHOULD BE ABLE TO:

1. Describe the just-in-case inventory management model.
2. Discuss just-in-time (JIT) inventory management
3. Explain the basic concepts of constrained optimization.
4. Define the theory of constraints, and tell how it can be used to manage inventory.

Excessive amounts of inventory can prove to be very costly. There are many ways to manage inventory costs, including the EOQ model, JIT, and the theory of constraints. All three methods offer ways of reducing inventory costs. The best approach usually depends on the nature of the organization as well as the nature of the inventory itself.

Inventory represents a significant investment of capital for most companies. Inventory ties up money that could be used more productively elsewhere. Thus, effective inventory management offers the potential for significant cost savings. Furthermore, quality, product engineering, prices, overtime, excess capacity, ability to respond to customers (due-date performance), lead times, and overall profitability are all affected by inventory levels. For example, Bal Seal Engineering used the theory of constraints to reduce inventory by 50 percent and double profits. ${ }^{1}$

[^0]Describing how inventory policy can be used to reduce costs and help organizations strengthen their competitive position is the main purpose of this chapter. First, we review just-in-case inventory management-a traditional inventory model based on anticipated demand. Learning the basics of this model and its underlying conceptual foundation will help us understand where it can still be appropriately applied. Understanding just-in-case inventory management also provides the necessary background for grasping the advantages of inventory management methods that are used in the contemporary manufacturing environment. These methods include JIT and the theory of constraints. To fully appreciate the theory of constraints, a brief introduction to constrained optimization (linear programming) is also needed. Although the focus of this chapter is inventory management, the theory of constraints is much more than an inventory management technique, and so we also explore what is called constraint accounting.

## Just-in-Case Inventory Management

Inventory management is concerned with managing inventory costs. Three types of inventory costs can be readily identified with inventory: (1) the cost of acquiring inventory (other than the cost of the good itself), (2) the cost of holding inventory, and (3) the cost of not having inventory on hand when needed.

If the inventory is a material or good acquired from an outside source, then these inventory-acquisition costs are known as ordering costs. Ordering costs are the costs of placing and receiving an order. Examples include the costs of processing an order (clerical costs and documents), insurance for shipment, and unloading costs. If the material or good is produced internally, then the acquisition costs are called setup costs. Setup costs are the costs of preparing equipment and facilities so they can be used to produce a particular product or component. Examples are wages of idled production workers, the cost of idled production facilities (lost income), and the costs of test runs (labor, materials, and overhead). Ordering costs and setup costs are similar in nature-both represent costs that must be incurred to acquire inventory. They differ only in the nature of the prerequisite activity (filling out and placing an order versus configuring equipment and facilities). Thus, in the discussion that follows, any reference to ordering costs can be viewed as a reference to setup costs.

Carrying costs are the costs of holding inventory. Examples include insurance, inventory taxes, obsolescence, the opportunity cost of funds tied up in inventory, handling costs, and storage space.

If demand is not known with certainty, a third category of inventory costs-called stock-out costs-exists. Stock-out costs are the costs of not having a product available when demanded by a customer. Examples are lost sales (both current and future), the costs of expediting (increased transportation charges, overtime, and so on), and the costs of interrupted production.

## Justifying Inventory

Effective inventory management requires that inventory-related costs be minimized. Minimizing carrying costs favors ordering or producing in small lot sizes, whereas minimizing ordering costs favors large, infrequent orders (minimization of setup costs favors long, infrequent production runs). The need to balance these two sets of costs so that the total cost of carrying and ordering can be minimized is one reason organizations choose to carry inventory.

Demand uncertainty is a second major reason for holding inventory. If the demand for materials or products is greater than expected, inventory can serve as a buffer, giving organizations the ability to meet delivery dates (thus keeping customers satisfied). Although balancing conflicting costs and dealing with uncertainty are the two most frequently cited reasons for carrying inventories, other reasons exist.

Inventories of parts and materials are often viewed as necessary because of supply uncertainties. That is, inventory buffers of parts and materials are needed to keep production flowing in case of late deliveries or no deliveries. (Strikes, bad weather, and bankruptcy are examples of uncertain events that can cause an interruption in supply.) Unreliable production processes may also create a demand for producing extra inventory. For example, a company may decide to produce more units than needed to meet demand because the production process usually yields a large number of nonconforming units. Similarly, buffers of inventories may be required to continue supplying customers or processes with goods even if a process goes down because of a failed machine. Finally, organizations may acquire larger inventories than normal to take advantage of quantity discounts or to avoid anticipated price increases. Exhibit 21-1 summarizes the reasons typically offered for carrying inventory. It is important to realize that these reasons are given to justify carrying inventories. A host of other reasons can be offered that encourage the carrying of inventories. For example, performance measures such as measures of machine and labor efficiency may promote the buildup of inventories.

## EXHIBIT 21-1 Traditional Reasons for Carrying Inventory

1. To balance ordering or setup costs and carrying costs
2. Demand uncertainty
3. Machine failure
4. Defective parts
5. Unavailable parts
6. Late delivery of parts
7. Unreliable production processes
8. To take advantage of discounts
9. To hedge against future price increases

## Economic Order Quantity: A Model for Balancing Acquisition and Carrying Costs

Of the nine reasons for holding inventory listed in Exhibit 21-1, the first reason is directly concerned with the trade-off between acquisition and carrying costs. Most of the other reasons are concerned directly or indirectly with stock-out costs, with the exception of the last two (which are concerned with managing the cost of the good itself). Initially, we will assume away the stock-out cost problem and focus only on the objective of balancing acquisition costs with carrying costs. To develop an inventory policy that deals with the trade-offs between these two costs, two basic questions must be addressed:

1. How much should be ordered (or produced) to minimize inventory costs?
2. When should the order be placed (or the setup done)?

The first question needs to be addressed before the second can be answered.

## Minimizing Total Ordering and Carrying Costs

Assuming that demand is known, the total ordering (or setup) and carrying cost can be described by the following equation:

$$
\begin{align*}
T C & =P D / Q+C Q / 2  \tag{21.1}\\
& =\text { Ordering (or setup) cost }+ \text { Carrying cost }
\end{align*}
$$

where
$T C=$ The total ordering (or setup) and carrying cost
$P=$ The cost of placing and receiving an order (or the cost of setting up a production run)
$Q=$ The number of units ordered each time an order is placed (or the lot size for production)
$D=$ The known annual demand
$C=$ The cost of carrying one unit of stock for one year
The cost of carrying inventory can be computed for any organization that carries inventories, although the inventory cost model using setup costs and lot size as inputs pertains only to manufacturers. To illustrate Equation 21.1 , consider Mantener Corporation, a service organization that does warranty work for a major producer of video recorders. Assume that the following values apply for a part used in the repair of the video recorders (the part is purchased from external suppliers):

$$
\begin{aligned}
& D=25,000 \text { units } \\
& Q=500 \text { units } \\
& P=\$ 40 \text { per order } \\
& C=\$ 2 \text { per unit }
\end{aligned}
$$

The number of orders per year is $D / Q$ which is $50(25,000 / 500)$. Multiplying the number of orders per year by the cost of placing and receiving an order $(D / Q \times$ $P)$ yields the total ordering cost of $\$ 2,000(50 \times \$ 40)$.

Carrying cost for the year is $C Q / 2$, which is simply the average inventory on hand $(Q / 2)$ multiplied by the carrying cost per unit $(C)$. (Assuming average inventory to be $Q / 2$ is equivalent to assuming that inventory is consumed uniformly.) For our example, the average inventory is $250(500 / 2)$ and the carrying cost for the year is $\$ 500$ $(\$ 2 \times 250)$. Applying Equation 21.1, the total cost is $\$ 2,500(\$ 2,000+\$ 500)$. An order quantity of 500 with a total cost of $\$ 2,500$, however, may not be the best choice. Some other order quantity may produce a lower total cost. The objective is to find the order quantity that minimizes the total cost, known as the economic order quantity (EOQ). The EOQ model is an example of a just-in-case or push inventory system. In a push system, the acquisition of inventory is initiated in anticipation of future demandnot in reaction to present demand. Fundamental to the analysis is the assessment of $D$, the future demand.

## Calculating EOQ

The decision variable for Equation 21.1 is the order quantity (or lot size). We seek the quantity that minimizes the total cost expressed by Equation 21.1. This quantity is the economic order quantity and is derived by taking the first derivative of Equation 21.1 with respect to $Q$ and solving for $Q:^{2}$

$$
\begin{equation*}
Q=E O Q=\sqrt{(2 D P / C)} \tag{21.2}
\end{equation*}
$$

The data of the preceding example are used to illustrate the calculation of EOQ using Equation 21.2:

$$
\begin{aligned}
E O Q & =\sqrt{(2 \times 25,000 \times \$ 40) / \$ 2} \\
& =\sqrt{1,000,000} \\
& =1,000
\end{aligned}
$$

Substituting 1,000 as the value of $Q$ in Equation 21.1 yields a total cost of $\$ 2,000$. The number of orders placed would be $25(25,000 / 1,000)$; thus, the total ordering cost is $\$ 1,000(25 \times \$ 40)$. The average inventory is $500(1,000 / 2)$, with a total carrying cost

[^1]of $\$ 1,000(500 \times \$ 2)$. Notice that the carrying cost equals the ordering cost. This is always true for the simple EOQ model described by Equation 21.2. Also, notice that an order quantity of 1,000 is less costly than an order quantity of 500 ( $\$ 2,000$ versus $\$ 2,500$ ).

## When to Order or Produce

Not only must we know how much to order (or produce) but we also must know when to place an order (or to set up for production). Avoiding stock-out costs is a key element in determining when to place an order. The reorder point is the point in time when a new order should be placed (or setup started). It is a function of the EOQ, the lead time, and the rate at which inventory is depleted. Lead time is the time required to receive the economic order quantity once an order is placed or a setup is initiated.

To avoid stock-out costs and to minimize carrying costs, an order should be placed so that it arrives just as the last item in inventory is used. Knowing the rate of usage and lead time allows us to compute the reorder point that accomplishes these objectives:

$$
\begin{equation*}
\text { Reorder point }=\text { Rate of usage } \times \text { Lead time } \tag{21.3}
\end{equation*}
$$

To illustrate Equation 21.3, we will continue to use the video recorder example. Assume that the repair activity uses 100 parts per day and that the lead time is four days. If so, an order should be placed when the inventory level of the VCR part drops to 400 units $(100 \times 4)$. Exhibit $21-2$ provides a graphical illustration. Note that the inventory is depleted just as the order arrives and that the quantity on hand jumps back up to the EOQ level.


## Demand Uncertainty and Reordering

If the demand for the part or product is not known with certainty, the possibility of stock-out exists. For example, if the VCR part was used at a rate of 120 parts a day
instead of 100 , the firm would use 400 parts after three and one-third days. Since the new order would not arrive until the end of the fourth day, repair activity requiring this part would be idled for two-thirds of a day. To avoid this problem, organizations often choose to carry safety stock. Safety stock is extra inventory carried to serve as insurance against fluctuations in demand. Safety stock is computed by multiplying the lead time by the difference between the maximum rate of usage and the average rate of usage. For example, if the maximum usage of the VCR part is 120 units per day, the average usage is 100 units per day, and the lead time is four days, then the safety stock is computed as follows:

| Maximum usage | 120 |
| :--- | :---: |
| Average usage | $\underline{(100})$ |
| Difference | $\underline{\times 4}$ |
| Lead time | $\underline{80}$ |
| Safety stock |  |

With the presence of safety stock, the reorder point is computed as follows:
Reorder point $=($ Average rate of usage $\times$ Lead time $)+$ Safety stock
For the repair service example, the reorder point with safety stock is computed as follows:

$$
\begin{aligned}
\text { Reorder point } & =(100 \times 4)+80 \\
& =480 \text { units }
\end{aligned}
$$

Thus, an order is automatically placed whenever the inventory level drops to 480 units.

## An Example Involving Setups

The same inventory management concepts apply to settings where inventory is manufactured. To illustrate, consider Expedition Company, a large manufacturer of garden and lawn equipment. One large plant in Kansas produces edgers. The manager of this plant is trying to determine the size of the production runs for the edgers. He is convinced that the current lot size is too large and wants to identify the quantity that should be produced to minimize the sum of the carrying and setup costs. He also wants to avoid stock-outs, since any stock-out would cause problems with the plant's network of retailers.

To help him in his decision, the controller has supplied the following information:
Average demand for edgers: 720 per day
Maximum demand for edgers: 780 per day
Annual demand for edgers: 180,000
Unit carrying cost: $\$ 4$
Setup cost: \$10,000
Lead time: 22 days
Based on the preceding information, the economic order quantity and the reorder point are computed in Exhibit 21-3. As the computation illustrates, the edgers should be produced in batches of 30,000 , and a new setup should be started when the supply of edgers drops to 17,160 .

## EOQ and Inventory Management

The traditional approach to managing inventory has been referred to as a just-in-case system. ${ }^{3}$ In some settings, a just-in-case inventory system is entirely appropriate. For ex-

[^2]
## EXHIBIT 21-3 EOQ and Reorder Point Illustrated

$$
\begin{aligned}
\mathrm{EOQ} & =\sqrt{2 \mathrm{DP} / \mathrm{C}} \\
& =\sqrt{(2 \times 180,000 \times \$ 10,000) / \$ 4} \\
& =\sqrt{900,000,000} \\
& =30,000 \text { edgers }
\end{aligned}
$$

Safety stock:

| Maximum usage | 780 |
| :--- | :---: |
| Average usage | $\underline{(720)}$ |
| Difference | $\underline{60}$ |
| Lead time | $\underline{\underline{1,320}}$ |
| Safety stock |  |

$$
\begin{aligned}
\text { Reorder point } & =(\text { Average usage } \times \text { Lead time })+\text { Safety stock } \\
& =(720 \times 22)+1,320 \\
& =17,160 \text { edgers }
\end{aligned}
$$

ample, hospitals need inventories of medicines, drugs, and other critical supplies on hand at all times so that life-threatening situations can be handled. Using an economic order quantity coupled with safety stock would seem eminently sensible in such an environment. Relying on a critical drug to arrive just in time to save a heart attack victim is simply not practical. Furthermore, many smaller retail stores, manufacturers, and services may not have the buying power to command alternative inventory management systems such as just-in-time purchasing.

As the edger example illustrates (Exhibit 21-3), the EOQ model is very useful in identifying the optimal trade-off between inventory carrying costs and setup costs. It also is useful in helping to deal with uncertainty by using safety stock. The historical importance of the EOQ model in many American industries can be better appreciated by understanding the nature of the traditional manufacturing environment. This environment has been characterized by the mass production of a few standardized products that typically have a very high setup cost. The production of the edgers fits this pattern. The high setup cost encouraged a large batch size: 30,000 units. The annual demand of 180,000 units can be satisfied using only six batches. Thus, production runs for these firms tended to be quite long. Furthermore, diversity was viewed as being costly and was avoided. Producing variations of the product can be quite expensive, especially since additional, special features would usually demand even more expensive and frequent setups - the reason for the standardized products.


## JIT Inventory Management

The manufacturing environment for many of these traditional, large-batch, high-setupcost firms has changed dramatically in the past 10 to 20 years. For one thing, the competitive markets are no longer defined by national boundaries. Advances in transportation and communication have contributed significantly to the creation of global competition. Advances in technology have contributed to shorter life cycles for products, and product diversity has increased. Foreign firms offering higher-quality, lower-cost products with specialized features have created tremendous pressures for our domestic largebatch, high-setup-cost firms to increase both quality and product diversity while simultaneously reducing total costs. These competitive pressures have led many firms to abandon the EOQ model in favor of a JIT approach. JIT has two strategic objectives: to increase profits and to improve a firm's competitive position. These two objectives
are achieved by controlling costs (enabling better price competition and increased profits), improving delivery performance, and improving quality. JIT offers increased cost efficiency and simultaneously has the flexibility to respond to customer demands for better quality and more variety. Quality, flexibility, and cost efficiency are foundational principles for world-class competition.

Just-in-time inventory management represents the continual pursuit of productivity through the elimination of waste. Non-value-added activities are a major source of waste. From Chapter 12, we know that non-value-added activities are either unnecessary or necessary, but inefficient and improvable. Necessary activities are essential to the business and/or are of value to customers. Eliminating non-value-added activities is a major thrust of JIT, but it is also a basic objective of any company following the path of continuous improvement-regardless of whether or not JIT is being used.

Clearly, JIT is much more than an inventory management system. Inventories, however, are particularly viewed as representing waste. They tie up resources such as cash, space, and labor. They also conceal inefficiencies in production and increase the complexity of a firm's information system. Thus, even though JIT focuses on more than inventory management, control of inventory is an important ancillary benefit. In this chapter, the inventory dimension of JIT is emphasized. In Chapter 11, other benefits and features of JIT were described. Chapter 12, in particular, focused on nonvalue-added activity analysis.

## A Pull System

JIT is a manufacturing approach that maintains that goods should be pulled through the system by present demand rather than pushed through the system on a fixed schedule based on anticipated demand. Many fast-food restaurants, like Burger King, use a pull system to control their finished goods inventory. When a customer orders a hamburger, it is taken from the rack. When the number of hamburgers gets too low, the cooks make new hamburgers. Customer demand pulls the materials through the system. This same principle is used in manufacturing settings. Each operation produces only what is necessary to satisfy the demand of the succeeding operation. The material or subassembly arrives just in time for production to occur so that demand can be met.

One effect of JIT is to reduce inventories to very low levels. The pursuit of insignificant levels of inventories is vital to the success of JIT. This idea of pursuing insignificant inventories, however, necessarily challenges the traditional reasons for holding inventories (see Exhibit 21-1). These reasons are no longer viewed as valid.

According to the traditional view, inventories solve some underlying problem related to each of the reasons listed in Exhibit 21-1. For example, the problem of resolving the conflict between ordering or setup costs and carrying costs is solved by selecting an inventory level that minimizes the sum of these costs. If demand is greater than expected or if production is reduced by breakdowns and production inefficiencies, then inventories serve as buffers, providing customers with products that otherwise might not have been available. Similarly, inventories can prevent stock-outs caused by late delivery of material, defective parts, and failures of machines used to produce subassemblies. Finally, inventories are often the solution to the problem of buying the best materials for the least cost through the use of quantity discounts.

JIT refuses to use inventories as the solution to these problems. In fact, the JIT approach can be seen as substituting information for inventories. Companies must track materials and finished goods more carefully. To do that, the logistics industry has gone high-tech. Schneider National Company, a logistics firm, uses satellite tracking to tell a customer just where a particular shipment is and when it will be delivered. In an example of partnering, Schneider engineers assisted client PPG Industries by showing its Pennsylvania plant employees how to use the shipping and receiving facilities more efficiently. ${ }^{4}$

[^3]JIT inventory management offers alternative solutions that do not require high inventories.

## Setup and Carrying Costs: The JIT Approach

JIT takes a radically different approach to minimizing total carrying and setup costs. The traditional approach accepts the existence of setup costs and then finds the order quantity that best balances the two categories of costs. JIT, on the other hand, does not accept setup costs (or ordering costs) as a given; rather, JIT attempts to drive these costs to zero. If setup costs and ordering costs become insignificant, the only remaining cost to minimize is carrying cost, which is accomplished by reducing inventories to very low levels. This approach explains the push for zero inventories in a JIT system.

## Long-Term Contracts, Continuous Replenishment, and Electronic Data Interchange

Ordering costs are reduced by developing close relationships with suppliers. Negotiating long-term contracts for the supply of outside materials will obviously reduce the number of orders and the associated ordering costs. Retailers have found a way to reduce ordering costs by adopting an arrangement known as continuous replenishment. Continuous replenishment means a manufacturer assumes the inventory management function for the retailer. The manufacturer tells the retailer when and how much stock to reorder. The retailer reviews the recommendation and approves the order if it makes sense. Wal-Mart and Procter \& Gamble, for example, use this arrangement. ${ }^{5}$ The arrangement has reduced inventories for Wal-Mart and has also reduced stock-out problems. Additionally, Wal-Mart often sells Procter \& Gamble's goods before it has to pay for them. Procter \& Gamble, on the other hand, has become a preferred supplier, has more and better shelf space, and also has less demand uncertainty. The ability to project demand more accurately allows Procter \& Gamble to produce and deliver continuously in smaller lots-a goal of JIT manufacturing. Similar arrangements can be made between manufacturers and suppliers.

The process of continuous replenishment is facilitated by electronic data interchange. Electronic data interchange (EDI) allows suppliers access to a buyer's online database. By knowing the buyer's production schedule (in the case of a manufacturer), the supplier can deliver the needed parts where they are needed just in time for their use. EDI involves no paper-no purchase orders or invoices. The supplier uses the production schedule, which is in the database, to determine its own production and delivery schedules. When the parts are shipped, an electronic message is sent from the supplier to the buyer that a shipment is en route. When the parts arrive, a bar code is scanned with an electronic wand, and this initiates payment for the goods. Clearly, EDI requires a close working arrangement between the supplier and the buyer-they almost operate as one company rather than two separate companies.

## Reducing Setup Times

Reducing setup times requires a company to search for new, more efficient ways to accomplish setup. Fortunately, experience has indicated that dramatic reductions in setup times can be achieved. A classic example is that of Harley-Davidson. Upon adopting a JIT system, Harley-Davidson reduced setup time by more than 75 percent on the machines evaluated. ${ }^{6}$ In some cases, Harley-Davidson was able to reduce the setup times from hours to minutes. Other companies have experienced similar results. Generally, setup times can be reduced by at least 75 percent.

[^4]
## Due-Date Performance: The JIT Solution

Due-date performance is a measure of a firm's ability to respond to customer needs. In the past, finished goods inventories have been used to ensure that a firm is able to meet a requested delivery date. JIT solves the problem of due-date performance not by building inventory but by dramatically reducing lead times. Shorter lead times increase a firm's ability to meet requested delivery dates and to respond quickly to the demands of the market. Thus, the firm's competitiveness is improved. JIT cuts lead times by reducing setup times, improving quality, and using cellular manufacturing.

Manufacturing cells reduce travel distance between machines and inventory; they can also have a dramatic effect on lead time. For example, in a traditional manufacturing system, one company took two months to manufacture a valve. By grouping the lathes and drills used to make the valves into U-shaped cells, the lead time was reduced to two or three days. A chain saw manufacturer was able to reduce travel distance from 2,620 feet to 173 feet and lead times from 21 days to three. Because of the reduced lead time and plans for even further reduction, the company will be filling orders directly from the factory rather than from finished goods warehouses. ${ }^{7}$ These reductions in lead time are not unique-most companies experience at least a 90 percent reduction in lead times when they implement JIT. ${ }^{8}$

Manufacturers are not the only companies using a JIT approach to improve time to market. Benetton calls itself an apparel services company, not a retailer. Operating one giant distribution center in Castrette, Italy, Benetton uses robots to send the latest fashions to any of its company stores in 120 countries within 12 days.

## Avoidance of Shutdown and Process Reliability: The JIT Approach

Most shutdowns occur for one of three reasons: machine failure, defective material or subassembly, and unavailability of a material or subassembly. Holding inventories is one solution to all three problems.

Those espousing the JIT approach claim that inventories do not solve the problems but cover up or hide them. JIT proponents use the analogy of rocks in a lake. The rocks represent the three problems, and the water represents inventories. If the lake is deep (inventories are high), then the rocks are never exposed, and managers can pretend they do not exist. By reducing inventories to zero, the rocks are exposed and can no longer be ignored. JIT solves the three problems by emphasizing total preventive maintenance and total quality control in addition to building the right kind of relationship with suppliers.

## Total Preventive Maintenance

Zero machine failures is the goal of total preventive maintenance. By paying more attention to preventive maintenance, most machine breakdowns can be avoided. This objective is easier to attain in a JIT environment because of the interdisciplinary labor philosophy. It is fairly common for a cell worker to be trained in maintenance of the machines he or she operates. Because of the pull-through nature of JIT, cell workers may have idle manufacturing time. Some of this time, then, can be used productively by having the cell workers involved in preventive maintenance.

## Total Quality Control

The problem of defective parts is solved by striving for zero defects. Because JIT manufacturing does not rely on inventories to replace defective parts or materials, the emphasis on quality for both internally produced and externally purchased materials

[^5]increases significantly. The outcome is impressive: the number of rejected parts tends to fall by 75-90 percent. Decreasing defective parts also diminishes the justification for inventories based on unreliable processes.

## The Kanban System

To ensure that parts or materials are available when needed, a system called the Kanban system is employed. This is an information system that controls production through the use of markers or cards. The Kanban system is responsible for ensuring that the necessary products (or parts) are produced (or acquired) in the necessary quantities at the necessary time. It is the heart of the JIT inventory management system.

A Kanban system uses cards or markers, which are plastic, cardboard, or metal plates measuring four inches by eight inches. The Kanban is usually placed in a vinyl sack and attached to the part or a container holding the needed parts.

A basic Kanban system uses three cards: a withdrawal Kanban, a production Kanban, and a vendor Kanban. The first two control the movement of work among the manufacturing processes, while the third controls movement of parts between the processes and outside suppliers. A withdrawal Kanban specifies the quantity that a subsequent process should withdraw from the preceding process. A production Kanban specifies the quantity that the preceding process should produce. Vendor Kanbans are used to notify suppliers to deliver more parts; they also specify when the parts are needed. The three Kanbans are illustrated in Exhibits 21-4, 21-5, and 21-6 (on the following page), respectively.

How Kanban cards are used to control the work flow can be illustrated with a simple example. Assume that two processes are needed to manufacture a product. The first process (CB Assembly) builds and tests printed circuit boards (using a U-shaped manufacturing

| EXHIBIT 21-4 | Withdrawal Kanban |  |
| :--- | :---: | :---: |
| Item No. |  |  |
| Item Name | 15670T07 |  |
| Computer Type | Circuit Board | Preceding Process |
| Box Capacity | TR6547 PC | CB Assembly |
| Box Type | $\mathbf{8}$ |  |


| EXHIBIT 21-5 | Production Kanban |  |
| :--- | :---: | :---: |
| Item No. $\frac{15670 \mathrm{~T} 07}{}$ |  |  |
| Item Name | Circuit Board | Preceding Process |
| Computer Type | TR6547 PC |  |
| Box Capacity | $\mathbf{8}$ |  |
| Box Type | C |  |


cell). The second process (Final Assembly) puts eight circuit boards into a subassembly purchased from an outside supplier. The final product is a personal computer.

Exhibit 21-7 provides the plant layout corresponding to the manufacture of the personal computers. Refer to the exhibit as the steps involved in using Kanbans are outlined.


Consider first the movement of work between the two processing areas. Assume that eight circuit boards are placed in a container and that one such container is located in the CB stores area. Attached to this container is a production Kanban (P-Kanban). A second container with eight circuit boards is located near the Final Assembly line (the withdrawal store) with a withdrawal Kanban (W-Kanban). Now assume that the production schedule calls for the immediate assembly of a computer.

The Kanban setups can be described as follows:

1. A worker from the Final Assembly line goes to the withdrawal store, removes the eight circuit boards, and places them into production. The worker also removes the withdrawal Kanban and places it on the withdrawal post.
2. The withdrawal Kanban on the post signals that the Final Assembly unit needs an additional eight circuit boards.
3. A worker from Final Assembly (or a material handler called a carrier) removes the withdrawal Kanban from the post and carries it to CB stores.
4. At the CB stores area, the carrier removes the production Kanban from the container of eight circuit boards and places it on the production ordering post.
5. The carrier next attaches the withdrawal Kanban to the container of parts and carries the container back to the Final Assembly area. Assembly of the next computer can begin.
6. The production Kanban on the production ordering post signals the workers of CB Assembly to begin producing another lot of circuit boards. The production Kanban is removed and accompanies the units as they are produced.
7. When the lot of eight circuit boards is completed, the units are placed in a container in the CB stores area with the production Kanban attached. The cycle is then repeated.
The use of Kanbans ensures that the subsequent process (Final Assembly) withdraws the circuit boards from the preceding process (CB Assembly) in the necessary quantity at the appropriate time. The Kanban system also controls the preceding process by allowing it to produce only the quantities withdrawn by the subsequent process. In this way, inventories are kept at a minimum, and the components arrive just in time to be used.

Essentially, the same steps are followed for a purchased subassembly. The only difference is the use of a vendor Kanban in place of a production Kanban. A vendor Kanban on a vendor post signals to the supplier that another order is needed. As with the circuit boards, the subassemblies must be delivered just in time for use. A JIT purchasing system requires the supplier to deliver small quantities on a frequent basis. These deliveries could be weekly, daily, or even several times a day. This calls for a close working relationship with suppliers. Long-term contractual agreements tend to ensure supply of materials.

## Discounts and Price Increases: JIT Purchasing versus Holding Inventories

Traditionally, inventories are carried so that a firm can take advantage of quantity discounts and hedge against future price increases of the items purchased. The objective is to lower the cost of inventory. JIT achieves the same objective without carrying inventories. The JIT solution is to negotiate long-term contracts with a few chosen suppliers located as close to the production facility as possible and to establish more extensive supplier involvement. Suppliers are not selected on the basis of price alone. Performancethe quality of the component and the ability to deliver as needed-and commitment to JIT purchasing are vital considerations. Other benefits of long-term contracts exist. They stipulate prices and acceptable quality levels. Long-term contracts also reduce dramatically the number of orders placed, which helps to drive down the ordering cost. Another effect of JIT purchasing is to lower the cost of purchased parts by 5 to 20 percent. ${ }^{9}$

## JIT's Limitations

JIT is not simply an approach that can be purchased and plugged in with immediate results. Its implementation should be more of an evolutionary process than a revolutionary process. Patience is needed. JIT is often referred to as a program of simplification-yet, this does not imply that it is simple or easy to implement. Time is required, for example, to build sound relationships with suppliers. Insisting on immediate changes in

[^6]
## COSTMANAGEMENT

## Technology in Action

Mercedes-Benz U.S. International (MBUSI) produces an M-Class SUV in its Tuscaloosa, Alabama, plant. The plant produces a variety of models, including V6, V8, 4 -cylinder, and left- and right-hand versions. The plant uses a JIT purchasing and manufacturing system to build the SUVs. The plant uses radio frequency identification (RFID) tags to ensure that materials are delivered on time to the production line. An RFID tag is placed on the vehicle at the beginning of production. When the vehicle
reaches a certain stage of production, a broadcast is sent to one of six sequence suppliers. The supplier builds the needed part and delivers it to the point in the production line just as it is needed. The RFID technology is also used to communicate to the suppliers whether the Tuscaloosa plant is running fast, slow, or normal, thus helping them with their daily production planning. In other words, RFID tags serve as an automated version of the vendor Kanbans.

Source: Ken Krizner, "Daffron, Andy-Interviews," Frontline Solutions, Vol. 1, Issue 9 (August 2000): 9.
delivery times and quality may not be realistic and may cause difficult confrontations between a company and its suppliers. Partnership, not coercion, should be the basis of supplier relationships. To achieve the benefits that are associated with JIT purchasing, a company may be tempted to redefine unilaterally its supplier relationships. Unilaterally redefining supplier relationships by extracting concessions and dictating terms may create supplier resentment and actually cause suppliers to retaliate. In the long run, suppliers may seek new markets, find ways to charge higher prices (than would exist with a preferred supplier arrangement), or seek regulatory relief. These actions may destroy many of the JIT benefits extracted by the impatient company.

Workers also may be affected by JIT. Studies have shown that sharp reductions in inventory buffers may cause a regimented work flow and high levels of stress among production workers. Some have suggested a deliberate pace of inventory reduction to allow workers to develop a sense of autonomy and to encourage their participation in broader improvement efforts. Forced and dramatic reductions in inventories may indeed reveal problems-but it may cause more problems: lost sales and stressed workers. If the workers perceive JIT as a way of simply squeezing more out of them, then JIT efforts may be doomed. Perhaps a better strategy for JIT implementation is one where inventory reductions follow the process improvements that JIT offers. Implementing JIT is not easy; it requires careful and thorough planning and preparation. Companies should expect some struggle and frustration.

The most glaring deficiency of JIT is the absence of inventory to buffer production interruptions. Current sales are constantly being threatened by an unexpected interruption in production. In fact, if a problem occurs, JIT's approach consists of trying to find and solve the problem before any further production activity occurs. Retailers who use JIT tactics also face the possibility of shortages. JIT retailers order what they need now-not what they expect to sell—because the idea is to flow goods through the channel as late as possible, hence keeping inventories low and decreasing the need for markdowns. If demand increases well beyond the retailer's supply of inventory, the retailer may be unable to make order adjustments quickly enough to avoid irked customers and lost sales. For example, a dockworkers' strike at U.S. west coast docks during the fall of 2002 had a strong impact on the Christmas shopping season. Many retailers were affected as products ordered for delivery during the fall were locked up at the docks. Toys "R" Us saw shortages of "Hello Kitty" merchandise result in significant lost sales. Manufacturers also face problems with shortages. For example, NUMMI (the U.S.-based joint venture between GM and Toyota) had to shut down its Fremont, California, manufacturing plant due to shortages of imported engines and transmissions. Yet, in spite of the downside, many retailers and manufacturers seem to be strongly committed to JIT. Apparently, losing sales on occasion is less costly than carrying high levels of inventory.

Even so, we must recognize that a sale lost today is a sale lost forever. Installing a JIT system so that it operates with very little interruption is not a short-run project. Thus, losing sales is a real cost of installing a JIT system. An alternative, and perhaps complementary approach, is the theory of constraints (TOC). In principle, TOC can be used in conjunction with JIT manufacturing. After all, JIT manufacturing environments also have constraints. Furthermore, the TOC approach has the very appealing quality of protecting current sales while also striving to increase future sales by increasing quality, lowering response time, and decreasing operating costs. However, before we introduce and discuss the theory of constraints, we need to provide a brief introduction to constrained optimization theory.


## Basic Concepts of Constrained Optimization

Manufacturing and service organizations must choose the mix of products that they will produce and sell. Decisions about product mix can have a significant impact on an organization's profitability. Each mix represents an alternative that carries with it an associated profit level. A manager should choose the alternative that maximizes total profits. The usual approach is to assume that only unit-based variable costs are relevant to the product mix decision. Thus, assuming that non-unit-level costs are the same for different mixes of products, a manager needs to choose the mix alternative that maximizes total contribution margin.

If a firm possesses unlimited resources and the demand for each product being considered is unlimited, then the product mix decision is simple-produce an infinite number of each product. Unfortunately, every firm faces limited resources and limited demand for each product. These limitations are called constraints. External constraints are limiting factors imposed on the firm from external sources (such as market demand). Internal constraints are limiting factors found within the firm (such as machine or labor time availability). Although resources and demands may be limited, certain mixes may not meet all the demand or use all of the resources available to be used. Constraints whose limited resources are not fully used by a product mix are loose constraints. If, on the other hand, a product mix uses all of the limited resources of a constraint, then the constraint is a binding constraint.

Constrained optimization is choosing the optimal mix given the constraints faced by the firm. Assume, for example, that Schaller Company produces two types of machine parts: X and Y , with unit contribution margins of $\$ 300$ and $\$ 600$, respectively. Assuming that Schaller can sell all that is produced, some may argue that only Part Y should be produced and sold because it has the larger contribution margin. However, this solution is not necessarily the best. The selection of the optimal mix can be significantly affected by the relationships of the constrained resources to the individual products. These relationships affect the quantity of each product that can be produced and, consequently, the total contribution margin that can be earned. This point is most vividly illustrated with one binding internal resource constraint.

## One Binding Internal Constraint

Assume that each part must be drilled by a special machine. The firm owns three machines that together provide 120 drilling hours per week. Part X requires one hour of drilling, and Part Y requires three hours of drilling. Assuming no other binding constraints, what is the optimal mix of parts? Since each unit of Part X requires one hour of drilling, 120 units of Part X can be produced per week (120/1). At $\$ 300$ per unit, Schaller can earn a total contribution margin of $\$ 36,000$ per week. On the other hand, Part Y requires three hours of drilling per unit; therefore, forty $(120 / 3)$ parts can be produced. At $\$ 600$ per unit, the total contribution margin is $\$ 24,000$ per week. Producing only Part X yields a higher profit level than producing only Part Y-even though the unit contribution margin for Part Y is twice the amount of Part X.

The contribution margin per unit of each product is not the critical concern. The contribution margin per unit of scarce resource is the deciding factor. The product yielding the highest contribution margin per drilling hour should be selected. Part X earns $\$ 300$ per machine hour ( $\$ 300 / \mathrm{l}$ ), while Part Y earns only $\$ 200$ per machine hour $(\$ 600 / 3)$. Thus, the optimal mix is 120 units of Part X and none of Part Y, producing a total contribution margin of $\$ 36,000$ per week.

## Internal Binding Constraint and External Binding Constraint

The contribution margin per unit of scarce resource can also be used to identify the optimal product mix when a binding external constraint exists. For example, assume the same internal constraint of 120 drilling hours, but also assume that Schaller can sell at most 60 units of Part X and 100 units of Part Y. The internal constraint allows Schaller to produce 120 units of Part X, but this is no longer a feasible choice because only 60 units of X can be sold. Thus, we now have a binding external constraint-one that affects the earlier decision to produce and sell only Part X. Since the contribution per unit of scarce resource (machine hour) is \$300 for Part X and \$200 for Part Y, it still makes sense to produce as much of Part X as possible before producing any of Part Y. Schaller should first produce 60 units of Part X, using 60 machine hours. This leaves 60 machine hours, allowing the production of 20 units of Part Y. The optimal mix is now 60 units of Part X and 20 units of Part Y, producing a total contribution margin of $\$ 30,000$ per week $[(\$ 300 \times 60)+(\$ 600 \times 20)]$.

## Multiple Internal Binding Constraints

It is possible for an organization to have more than one binding constraint. All organizations face multiple constraints: limitations of materials, limitations of labor inputs, limited machine hours, and so on. The solution of the product mix problem in the presence of multiple internal binding constraints is considerably more complicated and requires the use of a specialized mathematical technique known as linear programming.

## Linear Programming

Linear programming is a method that searches among possible solutions until it finds the optimal solution. The theory of linear programming permits many solutions to be ignored. In fact, all but a finite number of solutions are eliminated by the theory, with the search then limited to the resulting finite set.

To illustrate how linear programming can be used to identify the optimal mix with multiple internally constrained resources, we will continue to use the Schaller Company example. However, the example will be expanded to include a wider variety of constraints. In addition to the constraints already identified, two more internal constraints will be added. Assume that the two parts ( X and Y ) are produced in three sequential processes: grinding, drilling, and polishing. The grinding process uses two machines that provide a total of 80 grinding hours per week. Each part requires one hour of grinding. The polishing process is labor intensive. This process provides 90 labor hours per week. Part X uses two hours per unit, and Part Y uses one hour per unit. Information on Schaller's constraints is summarized in Exhibit 21-8. As before, the objective is to maximize Schaller's total contribution margin subject to the constraints faced by Schaller.

The objective of maximizing total contribution margin can be expressed mathematically. Let $X$ be the number of units produced and sold of Part $X$, and let $Y$ stand for Part Y. Since the unit contribution margins are $\$ 300$ and $\$ 600$ for $X$ and $Y$, respectively, the total contribution margin $(Z)$ can be expressed as follows:

$$
\begin{equation*}
Z=\$ 300 X+\$ 600 Y \tag{21.5}
\end{equation*}
$$

Equation 21.5 is called the objective function, the function to be optimized.

| EXHIBIT 21-8 | Constraint Data: Schaller Company |  |  |
| :---: | :---: | :---: | :---: |
| Resource Name | Resource Available | Part X Resource Usage: per Unit | Part Y Resource Usage: per Unit |
| Grinding <br> Drilling <br> Polishing <br> Market demand: Part X <br> Market demand: Part Y | 80 grinding hours 120 drilling hours 90 labor hours 60 units 100 units | One hour One hour Two hours One unit Zero units | One hour Three hours One hour Zero units One unit |

Schaller also has five constraints. Using the information in Exhibit 21-8, the constraints are expressed mathematically as follows:

Internal constraints:

$$
\begin{align*}
X+Y & \leq 80  \tag{21.6}\\
X+3 Y & \leq 120  \tag{21.7}\\
2 X+Y & \leq 90 \tag{21.8}
\end{align*}
$$

External constraints:

$$
\begin{align*}
& X \leq 60  \tag{21.9}\\
& Y \leq 100 \tag{21.10}
\end{align*}
$$

Schaller's problem is to select the number of units of X and Y that maximize total contribution margin subject to the constraints in Equations 21.6-21.10. This problem can be expressed in the following way, which is the standard formulation for a linear programming problem (often referred to as a linear programming model):

$$
\operatorname{Max} Z=\$ 300 X+\$ 600 Y
$$

subject to

$$
\begin{aligned}
X+Y & \leq 80 \\
X+3 Y & \leq 120 \\
2 X+Y & \leq 90 \\
X & \leq 60 \\
Y & \leq 100 \\
X & \geq 0 \\
Y & \geq 0
\end{aligned}
$$

The last two constraints are called nonnegativity constraints and simply reflect the reality that negative quantities of a product cannot be produced. All constraints, taken together, are referred to as the constraint set.

A feasible solution is a solution that satisfies the constraints in the linear programming model. The collection of all feasible solutions is called the feasible set of solutions. For example, producing and selling one unit of Part X and one unit of Part Y would be a feasible solution and a member of the feasible set. This product mix clearly satisfies all constraints. But the mix would earn only $\$ 900$ per week. However, many feasible solutions offer higher profits (for example, producing two of each part). The objective is to identify the best. The best feasible solution-the one that maximizes the total contribution margin-is called the optimal solution.

## Graphical Solution

When there are only two products, the optimal solution can be identified by graphing. Since solving the problem by graphing provides considerable insight into the way linear programming problems are solved, the Schaller problem will be solved in this way. Four steps are followed in solving the problem graphically.

1. Graph each constraint.
2. Identify the feasible set of solutions.
3. Identify all corner-point values in the feasible set.
4. Select the corner point that yields the largest value for the objective function.

The graph of each constraint for the Schaller example is shown in Exhibit 21-9. The nonnegativity constraints put the graph in the first quadrant. The other constraints are graphed by assuming that equality holds. Since each constraint is a linear equation, the graph is obtained by identifying two points on the line, plotting those points, and connecting them.


A feasible area for each constraint (except for the nonnegativity constraints) is determined by everything that lies below (or to the left of) the resulting line. The feasible set or region is the intersection of each constraint's feasible area. The feasible set is shown by the figure $A B C D$ in the exhibit; it includes the boundary of the figure. Notice that only two of the five constraints qualify as candidates for binding constraints: the drilling and polishing constraints.

There are four corner points: $A, B, C$, and $D$. Their values, obtained directly from the graph, are $(0,0)$ for $A,(0,40)$ for $B,(30,30)$ for $C$, and $(45,0)$ for $D$. The impact of these values on the objective function is as follows (expressed in thousands):

| Corner Point | $X$-Value | $Y$-Value | $Z=\$ 300 X+\$ 600 Y$ |
| :---: | :---: | :---: | :---: |
| A | 0 | 0 | $\$$ |
| B | 0 | 40 | 24,000 |
| C | 30 | 30 | $27,000^{*}$ |
| D | 45 | 0 | 13,500 |

[^7]The optimal solution calls for producing and selling 30 units of Part X per week and 30 units of Part Y per week. No other feasible solution will produce a larger contribution margin. It has been shown in the literature on linear programming that the optimal solution will always be one of the corner points. Thus, once the graph is drawn and the corner points are identified, finding the solution is simply a matter of computing the value of each corner point and selecting the one with the greatest value.

Graphical solutions are not practical with more than two or three products. Fortunately, an algorithm called the simplex method can be used to solve larger linear programming problems. This algorithm has been coded and is available for use on computers to solve these larger problems.

The linear programming model is an important tool for making product mix decisions. Although the linear programming model produces an optimal product mix decision, its real managerial value-particularly in today's business environment-may be more related to the kinds of inputs that must be generated for the model to be used. Unit-level prices and unit-level variable costs must be assessed. Furthermore, applying the model forces management to identify internal and external constraints. Internal constraints relate to how products consume resources; thus, resource usage relationships must be identified. Once the constrained relationships are known to management, they can be used by management to identify ways of improving a firm's performance in a variety of ways, including inventory management.

## Theory of Constraints

The goal of the theory of constraints is to make money now and in the future by managing constraints. The theory of constraints (TOC) recognizes that the performance of any organization (system) is limited by its constraints. In operational terms, every system has at least one constraint that limits its output. The theory of constraints develops a specific approach to manage constraints to support the objective of continuous improvement. TOC, however, focuses on the system-level effects of continuous improvement. Each company (i.e., system) is compared to a chain. Every chain has a weakest link that may limit the performance of the chain as a whole. The weakest link is the system's constraint and is the key to improving overall organizational performance. Why? Ignoring the weakest link and improving any other link costs money and will not improve system performance. On the other hand, by strengthening the weakest link, system performance can be improved. At some point, however, strengthening the weakest link shifts the focus to a different link that has now become the weakest. This next-weakest link is now the key system constraint, and it must be strengthened so that overall system performance can be improved. Thus, TOC can be thought of as a systems approach to continuous improvement.

## Operational Measures

Given that the goal is to make money, TOC argues that the next crucial step is to identify operational measures that encourage achievement of the goal. TOC focuses on three operational measures of systems performance: throughput, inventory, and operating expenses. Throughput is the rate at which an organization generates money through sales. ${ }^{10}$ Operationally, throughput is the rate at which contribution dollars come into the organization. Thus, we have the following operational definition:

$$
\begin{equation*}
\text { Throughput }=(\text { Sales revenue }- \text { Unit-level variable expenses }) / \text { Time } \tag{21.11}
\end{equation*}
$$

Typically, the unit-level variable costs acknowledged are materials and power. Direct labor is viewed as a fixed unit-level expense and is not usually included in the definition.

[^8]With this understanding, throughput corresponds to contribution margin. It is also important to note that it is a global measure and not a local measure. Finally, throughput is a rate. It is the contribution earned per unit of time (per day, per month, etc.).

Inventory is all the money the organization spends in turning materials into throughput. In operational terms, inventory is money invested in anything that it intends to sell and, thus, expands the traditional definition to include assets such as facilities, equipment (which are eventually sold at the end of their useful lives), fixtures, and computers. In the TOC world, inventory is the money spent on items that do not have to be immediately expensed. Thus, inventory represents the money tied up inside the organization.

Operating expenses are defined as all the money the organization spends in turning inventories into throughput and, therefore, represent all other money that an organization spends. This includes direct labor and all operating and maintenance expenses. Thus, throughput is a measure of money coming into an organization, inventory measures the money tied up within the system, and operating expenses represent money leaving the system. Based on these three measures, the objectives of management can be expressed as increasing throughput, minimizing inventory, and decreasing operating expenses.

By increasing throughput, minimizing inventory, and decreasing operating expenses, the following three traditional financial measures of performance will be affected favorably: net income and return on investment will increase and cash flow will improve. Of the three TOC factors, throughput is viewed as being the most important for improving financial performance, followed by inventory, and then by operating expenses. The rationale for this order is straightforward. Operating expenses and inventories can be reduced at most to zero (inventory, though, being the larger amount), while there is virtually no upper limit on throughput. Increasing throughput and decreasing operating expenses have always been emphasized as key elements in improving the three financial measures of performance; the role of minimizing inventory, however, in achieving these improvements has been traditionally regarded as less important than reducing operating expenses.

The theory of constraints, like JIT, assigns inventory management a much more prominent role than does the traditional just-in-case viewpoint. TOC recognizes that lowering inventory decreases carrying costs and, thus, decreases operating expenses and improves net income. TOC, however, argues that lowering inventory helps produce a competitive edge by having better products, lower prices, and faster response to customer needs.

## Higher-Quality Products

Better products mean higher quality. It also means that the company is able to improve products and quickly provide these improved products to the market. The relationship between low inventories and quality has been described in the JIT section. Essentially, low inventories allow defects to be detected more quickly and the cause of the problem assessed.

Improving products is also a key competitive element. New or improved products need to reach the market quickly-before competitors can provide similar features. This goal is facilitated with low inventories. Low inventories allow new product changes to be introduced more quickly because the company has fewer old products (in stock or in process) that would need to be scrapped or sold before the new product is introduced.

## Lower Prices

High inventories mean more productive capacity is needed, leading to a greater investment in equipment and space. Since lead time and high work-in-process inventories are usually correlated, high inventories may often be the cause of overtime. Overtime, of course, increases operating expenses and lowers profitability. Lower inventories re-
duce carrying costs, per-unit investment costs, and other operating expenses such as overtime and special shipping charges. By lowering investment and operating costs, the unit margin of each product is increased, providing more flexibility in pricing decisions.

## Improved Delivery Performance

Delivering goods on time and producing goods with shorter lead times than the market dictates are important competitive tools. Delivering goods on time is related to a firm's ability to forecast the time required to produce and deliver goods. If a firm has higher inventories than its competitors, then the firm's production lead time is higher than the industry's forecast horizon. High inventories may obscure the actual time required to produce and fill an order. Lower inventories allow actual lead times to be more carefully observed, and more accurate delivery dates can be provided. Shortening lead times is also crucial. Shortening lead times is equivalent to lowering work-in-process inventories. A company carrying 10 days of work-in-process inventories has an average production lead time of 10 days. If the company can reduce lead time from 10 to five days, then the company should now be carrying only five days of work-in-process inventories. As lead times are reduced, it is also possible to reduce finished goods inventories. For example, if the lead time for a product is 10 days and the market requires delivery on demand, then the firms must carry, on average, 10 days of finished goods inventory (plus some safety stock to cover demand uncertainty). Suppose that the firm is able to reduce lead time to five days. In this case, finished goods inventory should also be reduced to five days. Thus, the level of inventories signals the organization's ability to respond. High levels relative to those of competitors translate into a competitive disadvantage. TOC, therefore, emphasizes reduction of inventories by reducing lead times.

## Five-Step Method for Improving Performance

The theory of constraints uses five steps to achieve its goal of improving organizational performance:

1. Identify an organization's constraints.
2. Exploit the binding constraints.
3. Subordinate everything else to the decisions made in step 2.
4. Elevate the organization's binding constraints.
5. Repeat the process as a new constraint emerges to limit output.

## Step 1: Identify an Organization's Constraints

Step 1 is identical in concept to the process described for linear programming. Internal and external constraints are identified. The optimal product mix is identified as the mix that maximizes throughput subject to all the organization's constraints. The optimal mix reveals how much of each constrained resource is used and which of the organization's constraints are binding.

## Step 2: Exploit the Binding Constraints

One way to make the best use of any binding constraints is to ensure that the optimal product mix is produced. Making the best use of binding constraints, however, is more extensive than simply ensuring production of the optimal mix. This step is the heart of TOC's philosophy of short-run constraint management and is directly related to TOC's goal of reducing inventories and improving performance.

Most organizations have only a few binding resource constraints. The major binding constraint is defined as the drummer. Assume, for example, that there is only one internal binding constraint. By default, this constraint becomes the drummer. The drummer constraint's production rate sets the production rate for the entire plant. Downstream processes fed by the drummer constraint are naturally forced to follow its rate of production. Scheduling for downstream processes is easy. Once a part is finished at the drummer process, the next process begins its operation. Similarly, each subsequent
operation begins when the prior operation is finished. Upstream processes that feed the drummer constraint are scheduled to produce at the same rate as the drummer constraint. Scheduling at the drummer rate prevents the production of excessive upstream work-in-process inventories.

For upstream scheduling, TOC uses two additional features in managing constraints to lower inventory levels and improve organizational performance: buffers and ropes. First, an inventory buffer is established in front of the major binding constraint. The inventory buffer is referred to as the time buffer. A time buffer is the inventory needed to keep the constrained resource busy for a specified time interval. The purpose of a time buffer is to protect the throughput of the organization from any disruption that can be overcome within the specified time interval. For example, if it takes one day to overcome most interruptions that occur upstream from the drummer constraint, then a 2 -day buffer should be sufficient to protect throughput from any interruptions. Thus, in scheduling, the operation immediately preceding the drummer constraint should produce the parts needed by the drummer resource two days in advance of their planned usage. Any other preceding operations are scheduled backwards in time to produce so that their parts arrive just in time for subsequent operations.

Ropes are actions taken to tie the rate at which material is released into the plant (at the first operation) to the production rate of the constrained resource. The objective of a rope is to ensure that the work-in-process inventory will not exceed the level needed for the time buffer. Thus, the drummer rate is used to limit the rate of material release and effectively controls the rate at which the first operation produces. The rate of the first operation then controls the rates of subsequent operations. The TOC inventory system is often called the drum-buffer-rope (DBR) system. Exhibit 21-10 illustrates the DBR structure for a general setting.

The Schaller Company example used to illustrate constrained optimization also can be used to provide a specific illustration of the DBR system. Recall that there are three sequential processes: grinding, drilling, and polishing. Each of these processes has a limited amount of resources. Demand for each type of machine part produced is also limited. However, from Exhibit 21-9 we know that the only binding constraints are the drilling and polishing constraints. We also know that the optimal mix consists of 30 units of Part X and 30 units of Part Y (per week). This is the most that the drilling and polishing processes can handle. Since the drilling process feeds the polishing process, we can define the drilling constraint as the drummer for the plant. Assume that the demand for each part is uniformly spread out over the week. This means that the production rate should be six per day of each part (for a 5 -day work week). A 2 -day time buffer would require 24 completed parts from the grinding process: 12 for Part X and 12 for Part Y. To ensure that the time buffer does not increase at a rate greater than six per day for each part, materials should be released to the grinding process such that only six of each part can be produced each day. (This is the rope-tying the release of materials to the production rate of the drummer constraint.) Exhibit 21-11, on page 952, summarizes the specific DBR details for the Schaller Company.

## Step 3: Subordinate Everything Else to the Decisions Made in Step 2

The drummer constraint essentially sets the capacity for the entire plant. All remaining departments should be subordinated to the needs of the drummer constraint. This principle requires many companies to change the way they view things. For example, the use of efficiency measures at the departmental level may no longer be appropriate. Consider the Schaller Company once again. Encouraging maximum productive efficiency for the grinding department would produce excess work-in-process inventories. The capacity of the grinding department is 80 units per week. Assuming the 2 -day buffer is in place, the grinding department would add 20 units per week to the buffer in front of the drilling department. Over a period of a year, the potential exists for building very large work-in-process inventories ( 1,000 units of the two parts would be added to the buffer over a 50 -week period).

## EXHIBIT 21-10 Drum-Buffer-Rope System: General Description



## Step 4: Elevate the Organization's Binding Constraint(s)

Once actions have been taken to make the best possible use of the existing constraints, the next step is to embark on a program of continuous improvement by reducing the limitations that the binding constraints have on the organization's performance. However, if there is more than one binding constraint, which one should be elevated? For example, in the Schaller Company setting, there are two binding constraints: the drilling constraint and the polishing constraint. In this case, the guideline is to increase the resource of the constraint that produces the greatest increase in throughput. To determine the most profitable effort, assume that one additional unit of resource is available for drilling (other resources are held constant), and then calculate the new optimal mix

and throughput. Now, repeat the process for the polishing constraint. Clearly, this approach can be tedious. Fortunately, the same information is produced as a by-product of the simplex method. The simplex method produces what are called shadow prices. Shadow prices indicate the amount by which throughput will increase for one additional unit of scarce resource. For the Schaller Company example, the shadow prices for the drilling and polishing resources are $\$ 180$ and $\$ 60$, respectively. Thus, Schaller should focus on busting the drilling constraint because it offers the most improvement.

Suppose, for example, that Schaller Company adds a half shift for the drilling department, increasing the drilling hours from 120 to 180 per week. Throughput will now be $\$ 37,800$, an increase of $\$ 10,800(\$ 180 \times 60$ additional hours). Furthermore, as you can check, the optimal mix is now 18 units of Part X and 54 units of Part Y. Is the half shift worth it? This question is answered by comparing the cost of adding the half shift with the increased throughput. If the cost is labor-say overtime at $\$ 50$ per hour (for all employees)-then the incremental cost is $\$ 3,000$, and the decision to add the half shift is a good one.

## Step 5: Repeat Process: Does a New Constraint Limit Throughput?

Eventually, the drilling resource constraint will be elevated to a point where the constraint is no longer binding. Suppose, for example, that the company adds a full shift
for the drilling operation, increasing the resource availability to 240 hours. The new constraint set is shown in Exhibit 21-12. Notice that the drilling constraint no longer affects the optimal mix decision. The grinding and polishing resource constraints are possible candidates for the new drummer constraint. Once the drummer constraint is identified, then the TOC process is repeated (step 5 ). The objective is to continually improve performance by managing constraints. Do not allow inertia to cause a new constraint. Focus now on the next-weakest link.

## EXHIBIT 21-12

New Constraint Set: Schaller Company


## System Improvement

The five steps just described can produce significant improvements in systems performance. Rockland Manufacturing, a producer of attachments for heavy construction equipment, made more profit in the two years following TOC implementation than in the previous 10 years. ${ }^{11}$ Rockland increased throughput, reduced work-in-process inventories, and achieved virtually a 100 percent on-time shipment rate. Similarly, Boeing's Printed Circuit Board Center, after three years of TOC, managed to reduce lead time by 75 percent, increase throughput by over 100 percent, and achieve significant improvement in on-time delivery of its products. ${ }^{12}$

[^9]
## S U M M A R Y

Three approaches to managing inventory were discussed: just-in-case, JIT, and theory of constraints. The traditional approach uses inventories to manage the trade-offs between ordering (setup) costs and carrying costs. The optimal trade-off defines the economic order quantity. Other reasons for inventories are also offered: due-date performance, avoiding shutdowns (protecting throughput), hedging against future price increases, and taking advantage of discounts. JIT and TOC, on the other hand, argue that inventories are costly and are used to cover up fundamental problems that need to be corrected so that the organization can become more competitive.

JIT uses long-term contracts, continuous replenishment, and EDI to reduce (eliminate) ordering costs. Engineering efforts are made to reduce setup times drastically. Once ordering costs and setup costs are reduced to minimal levels, then it is possible to reduce carrying costs by reducing inventory levels. JIT carries small buffers in front of each operation and uses a Kanban system to regulate production. Production is tied to market demand. If an interruption occurs, throughput tends to be lost because of the small buffers. Yet, future throughput tends to increase because efforts are made to improve such things as quality, productivity, and lead time.

TOC identifies an organization's constraints and exploits them so that throughput is maximized and inventories and operating costs are minimized. Identifying the optimal mix is part of this process. Linear programming is useful for this purpose. The major binding constraint is identified and is used to set the productive rate for the plant. Release of materials into the first process (operation) is regulated by the drummer constraint. A time buffer is located in front of critical constraints. This time buffer is sized so that it protects throughput from any interruptions. As in JIT, the interruptions are used to locate and correct the problem. However, unlike JIT, the time buffer serves to protect throughput. Furthermore, because buffers are located only in front of critical constraints, TOC may actually produce smaller inventories than JIT.

## REVIEW PROBLEMS AND SOLUTIONS

## 1 EOQ

Verijon, Inc., uses 15,000 pounds of plastic each year in its production of plastic cups. The cost of placing an order is $\$ 10$. The cost of holding one pound of plastic for one year is $\$ 0.30$. Verijon uses an average of 60 pounds of plastic per day. It takes five days to place and receive an order.

## Required:

1. Calculate the EOQ.
2. Calculate the annual ordering and carrying costs for the EOQ.
3. What is the reorder point?
4. $\mathrm{EOQ}=\sqrt{2 D P / C}$

$$
\begin{aligned}
& =\sqrt{(2 \times 15,000 \times \$ 10) / \$ 0.30} \\
& =\sqrt{1,000,000} \\
& =1,000
\end{aligned}
$$

2. Ordering cost $=(D / Q) P=(15,000 / 1,000) \$ 10=\$ 150$

Carrying cost $=(Q / 2) C=(1,000 / 2) \$ 0.30=\$ 150$
3. $\mathrm{ROP}=60 \times 5=300$ pounds (whenever inventory drops to this level, an order should be placed).

## 2 JIT, Drum-Buffer-Rope System

Both just-in-case and JIT inventory management systems have drummers-factors that determine the production rate of the plant. For a just-in-case system, the drummer is the excess capacity of the first operation. For JIT, the drummer is market demand.

## Required:

1. Explain why the drummer of a just-in-case system is identified as excess demand of the first operation.
2. Explain how market demand drives the JIT production system.
3. Explain how a drummer constraint is used in the TOC approach to inventory management.
4. What are the advantages and disadvantages of the three types of drummers?
5. In a traditional inventory system, local efficiency measures encourage the manager of the first operation to keep the department's workers busy. Thus, materials are released to satisfy this objective. This practice is justified because the inventory may be needed just in case demand is greater than expected, or just in case the first operation has downtime, etc.
6. In a JIT system, when the final operation delivers its goods to a customer, a backward rippling effect triggers the release of materials into the factory. First, the last process removes the buffer inventory from the withdrawal store, and this leads to a P-Kanban being placed on the production post of the preceding operation. This operation then begins production, withdrawing parts it needs from its withdrawal store, leading to a P-Kanban being placed on the production post of its preceding operation. This process repeats itself-all the way back to the first operation.
7. A drummer constraint sets the production rate of the factory to match its own production rate. This is automatically true for succeeding operations. For preceding operations, the rate is controlled by tying the drummer constraint's rate of production to that of the first operation. A time buffer is also set in front of the drummer constraint to protect throughput in the event of interruptions.
8. The excess capacity drummer typically will build excess inventories. This serves to protect current throughput. However, it ties up a lot of capital and tends to cover up problems such as poor quality, bad delivery performance, and inefficient production. Because it is costly and covers up certain critical productive problems, the just-in-case approach may be a threat to future throughput by damaging a firm's competitive position. JIT reduces inventories dramatically-using only small buffers in front of each operation as a means to regulate production flow and signal when production should occur. JIT has the significant advantage of uncovering problems and eventually correcting them. However, discovering problems usually means that current throughput will be lost while problems are being corrected. Future throughput tends to be protected because the firm is taking actions to improve its operations. TOC uses time buffers in front of the critical constraints. These buffers are large enough to keep the critical constraints operating while other operations may be down. Once the problem is corrected, the other resource constraints usually have sufficient excess capacity to catch up. Thus, current throughput is protected. Furthermore, future throughput is protected because TOC uses the same approach as JIT-namely, that of uncovering and correcting problems. TOC can be viewed as an improvement on JIT methodscorrecting the lost throughput problem while maintaining the other JIT features.

## KEY TERMS

Binding constraint 943
Carrying costs 930
Constrained optimization 943
Constraint set 945
Constraints 943
Continuous replenishment 937

Drum-buffer-rope (DBR) system 950
Drummer 949
Economic order quantity (EOQ) 932
Electronic data interchange (EDI) 937
External constraints 943
Feasible set of solutions 945

Feasible solution $945 \quad$ Reorder point 933
Internal constraints 943
Inventory 948
Just-in-case inventory management 930
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Lead time 933
Linear programming 944
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Objective function 944
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Production Kanban 939

Ropes 950
Safety stock 934
Setup costs 930
Shadow prices 952
Simplex method 947
Stock-out costs 930
Theory of constraints 947
Throughput 947
Time buffer 950
Total preventive maintenance 938
Vendor Kanbans 939
Withdrawal Kanban 939

## QUESTIONS FOR WRITING AND DISCUSSION

1. What are ordering costs? What are setup costs? What are carrying costs? Provide examples of each type of cost.
2. Explain why, in the traditional view of inventory, carrying costs increase as ordering costs decrease.
3. Discuss the traditional reasons for carrying inventory.
4. What are stock-out costs?
5. Explain how safety stock is used to deal with demand uncertainty.
6. What is the economic order quantity?
7. What approach does JIT take to minimize total inventory costs?
8. One reason for inventory is to prevent shutdowns. How does the JIT approach to inventory management deal with this potential problem?
9. Explain how the Kanban system helps reduce inventories.
10. Explain how long-term contractual relationships with suppliers can reduce the acquisition cost of materials.
11. What is a constraint? An internal constraint? An external constraint?
12. Explain the procedures for graphically solving a linear programming problem. What solution method is used when the problem includes more than two or three products?
13. Define and discuss the three measures of organizational performance used by the theory of constraints.
14. Explain how lowering inventory produces better products, lower prices, and better responsiveness to customer needs.
15. What are the five steps that TOC uses to improve organizational performance?

## EXERCISES

## 21-1 Ordering and Carrying Costs

LO1 Corsair, Inc., uses 40,000 plastic housing units each year in its production of paper shredders. The cost of placing an order is $\$ 40$. The cost of holding one unit of inventory for one year is $\$ 5$. Currently, Corsair places eight orders of 5,000 plastic housing units per year.

## Required:

l. Compute the annual ordering cost.
2. Compute the annual carrying cost.
3. Compute the cost of Corsair's current inventory policy. Is this the minimum cost? Why or why not?

## 21-2 Economic Order Quantity

LO1 Refer to the data in Exercise 21-1.


## Required:

1. Compute the economic order quantity.
2. Compute the ordering and carrying costs for the EOQ.
3. How much money does using the EOQ policy save the company over the policy of purchasing 5,000 plastic housing units per order?

## 21-3 Economic Order Quantity

LO1 Ulmer Company uses 312,500 pounds of sucrose each year. The cost of placing an order is $\$ 30$, and the carrying cost for one pound of sucrose is $\$ 0.75$.

## Required:

1. Compute the economic order quantity for sucrose.
2. Compute the carrying and ordering costs for the EOQ.

## 21-4 Reorder Point

LO1 Swann Company manufactures sleeping bags. A heavy-duty zipper is one part the company orders from an outside supplier. Information pertaining to the zipper is as follows:


| Economic order quantity | 4,200 units |
| :--- | ---: |
| Average daily usage | 200 units |
| Maximum daily usage | 240 units |
| Lead time | 3 days |

## Required:

1. What is the reorder point assuming no safety stock is carried?
2. What is the reorder point assuming that safety stock is carried?

## 21-5 EOQ with Setup Costs

LO1 Pawnee Manufacturing produces casings for stereo sets: large and small. In order to produce the different casings, equipment must be set up. Each setup configuration corresponds to a particular type of casing. The setup cost per production run-for either casing-is $\$ 6,000$. The cost of carrying small casings in inventory is $\$ 2$ per casing per year. The cost of carrying large casings is $\$ 6$ per year. To satisfy demand, the company produces 150,000 small casings and 50,000 large casings per year.

## Required:

1. Compute the number of small casings that should be produced per setup to minimize total setup and carrying costs for this product.
2. Compute the total setup and carrying costs associated with the economic order quantity for the small casings.

## 21-6 EOQ with Setup Costs

LO1 Refer to Exercise 21-5.

## Required:

1. Compute the number of large casings that should be produced per setup to minimize total setup and carrying costs for this product.
2. Compute the total setup and carrying costs associated with the economic order quantity for the large casings.

## 21-7 Reorder Point

LO1 Refer to Exercise 21-5. Assume the economic lot size for small casings is 30,000 and that of the large casings is 10,000 . Pawnee Manufacturing sells an average of 590 small casings per workday and an average of 200 large casings per workday. It takes Pawnee three days to set up the equipment for small or large casings. Once set up, it takes 20 workdays to produce a batch of small casings and 20 days for large casings. There are 250 workdays available per year.

## Required:

1. What is the reorder point for small casings? Large casings?
2. Using the economic order batch size, is it possible for Pawnee to produce the amount that can be sold of each casing? Does scheduling have a role here? Explain. Is this a push- or pull-through system approach to inventory management? Explain.

## 21-8 Safety Stock

LO1 Noble Manufacturing produces a component used in its production of clothes dryers. The time to set up and produce a batch of the components is two days. The average daily usage is 320 components, and the maximum daily usage is 375 components.

## Required:

Compute the reorder point assuming that safety stock is carried by Noble Manufacturing. How much safety stock is carried by Noble?

## 21-9 Kanban System, EDI

LO2 Hales Company produces a product that requires two processes. In the first process, a subassembly is produced (subassembly A). In the second process, this subassembly and a subassembly purchased from outside the company (subassembly B) are assembled to produce the final product. For simplicity, assume that the assembly of one final unit takes the same time as the production of subassembly A. Subassembly A is placed in a container and sent to an area called the subassembly stores (SB stores) area. A production Kanban is attached to this container. A second container, also with one subassembly, is located near the assembly line (called the withdrawal store). This container has attached to it a withdrawal Kanban.

## Required:

1. Explain how withdrawal and production Kanban cards are used to control the work flow between the two processes. How does this approach minimize inventories?
2. Explain how vendor Kanban cards can be used to control the flow of the purchased subassembly. What implications does this have for supplier relationships? What role, if any, do continuous replenishment and EDI play in this process?

## 21-1O JIT Limitations

LO2 Many companies have viewed JIT as a panacea-a knight in shining armor which promises rescue from sluggish profits, poor quality, and productive inefficiency. It is often lauded for its beneficial effects on employee morale and self-esteem. Yet, JIT may also cause a company to struggle and may produce a good deal of frustration. In some cases, JIT appears to deliver less than its reputation seems to call for.

## Required:

Discuss some of the limitations and problems that companies may encounter when implementing a JIT system.

## 21-1 1 Product Mix Decision, Single Constraint

 of pans requires the use of special machine that has total operating capacity of 182000 hours per year. Information on each of the three products is as follows:|  | Basic | Standard | Deluxe |
| :--- | ---: | :---: | ---: |
| Selling price | $\$ 12.00$ | $\$ 17.00$ | $\$ 32.00$ |
| Unit variable cost | $\$ 7.00$ | $\$ 11.00$ | $\$ 12.00$ |
| Machine hours required | 0.10 | 0.20 | 0.50 |

The marketing manager has determined that the company can sell all that it can produce of each of the three products.

## Required:

1. How many of each product should be sold to maximize total contribution margin? What is the total contribution margin for this product mix?
2. Suppose that Behar can sell no more than 300,000 units of each type at the prices indicated. What product mix would you recommend, and what would be the total contribution margin?

## 21-12 Drum-Buffer-Rope System

LO4 Duckstein, Inc., manufactures two types of aspirin: plain and buffered. It sells all it produces. Recently, Duckstein implemented a TOC approach for its Fort Smith plant. One binding constraint was identified, and the optimal product mix was determined. The diagram on the following page reflects the TOC outcome:

## Required:

1. What is the daily production rate? Which process sets this rate?
2. How many days of buffer inventory is Duckstein carrying? How is this time buffer determined?
3. Explain what the letters $\mathrm{A}, \mathrm{B}$, and C in the exhibit represent. Discuss each of their roles in the TOC system.


## PROBLEMS

## 21-13 EOQ, Safety Stock, Lead Time, Batch Size, and JIT

LO1, LO2 Bateman Company produces helmets for drivers of motorcycles. Helmets are produced in batches according to model and size. Although the setup and production time vary for each model, the smallest lead time is six days. The most popular model, Model HA2, takes two days for setup, and the production rate is 750 units per day. The expected annual demand for the model is 36,000 units. Demand for the model, however, can reach 45,000 units. The cost of carrying one HA2 helmet is $\$ 3$ per unit. The setup cost is $\$ 6,000$. Bateman chooses its batch size based on the economic order quantity criterion. Expected annual demand is used to compute the EOQ.

Recently, Bateman has encountered some stiff competition-especially from foreign sources. Some of the foreign competitors have been able to produce and deliver the helmets to retailers in half the time it takes Bateman to produce. For example, a large retailer recently requested a delivery of 12,000 Model HA2 helmets with the stipulation that the helmets be delivered within seven working days. Bateman had 3,000 units of HA2 in stock. Bateman informed the potential customer that it could deliver 3,000 units immediately and the other 9,000 units in about 14 working days-with the
possibility of interim partial orders being delivered. The customer declined the offer indicating that the total order had to be delivered within seven working days so that its stores could take advantage of some special local conditions. The customer expressed regret and indicated that it would accept the order from another competitor who could satisfy the time requirements.

## Required:

1. Calculate the optimal batch size for Model HA2 using the EOQ model. Was Bateman's response to the customer right? Would it take the time indicated to produce the number of units wanted by the customer? Explain with supporting computations.
2. Upon learning of the lost order, the marketing manager grumbled about Bateman's inventory policy. "We lost the order because we didn't have sufficient inventory. We need to carry more units in inventory to deal with unexpected orders like these." Do you agree or disagree? How much additional inventory would have been needed to meet customer requirements? In the future, should Bateman carry more inventory? Can you think of other solutions?
3. Fenton Gray, the head of industrial engineering, reacted differently to the lost order. "Our problem is more complex than insufficient inventory. I know that our foreign competitors carry much less inventory than we do. What we need to do is decrease the lead time. I have been studying this problem, and my staff have found a way to reduce setup time for Model HA2 from two days to 1.5 hours. Using this new procedure, setup cost can be reduced to about $\$ 94$. Also, by rearranging the plant layout for this product-creating what are called manufacturing cells-we can increase the production rate from 750 units per day to about 2,000 units per day. This is done simply by eliminating a lot of move time and waiting time-both non-value-added activities." Assume that the engineer's estimates are on target. Compute the new optimal batch size (using the EOQ formula). What is the new lead time? Given this new information, would Bateman have been able to meet the customer's time requirements? Assume that there are eight hours available in each workday.
4. Suppose that the setup time and cost are reduced to 0.5 hour and $\$ 10$, respectively. What is the batch size now? As setup time approaches zero and the setup cost becomes negligible, what does this imply? Assume, for example, that it takes five minutes to set up, and costs are about $\$ 0.864$ per setup.

## 21-14 Product Mix Decisions, Multiple Constraints

LO3 Cardin Company produces two types of gears: Model \#12 and Model \#15. Market conditions limit the number of each gear that can be sold. For Model \#12, no more than 15,000 units can be sold, and for Model \#15, no more than 40,000 units. Each gear must be notched by a special machine. Cardin owns eight machines that together provide 40,000 hours of machine time per year. Each unit of Model \#12 requires two hours of machine time, and each unit of Model \#15 requires one half hour of machine time. The unit contribution for Model \#l2 is \$30 and for Model \#15 is $\$ 15$. Cardin wants to identify the product mix that will maximize total contribution margin.

## Required:

1. Formulate Cardin's problem as a linear programming model.
2. Solve the linear programming model in Requirement 1.
3. Identify which constraints are binding and which are loose. Also, identify the constraints as internal or external.

## 21-15 Product Mix Decision, Single and Multiple Constraints

LO3 Taylor Company produces two industrial cleansers that use the same liquid chemical input: Pocolimpio and Maslimpio. Pocolimpio uses two quarts of the chemical for every unit produced, and Maslimpio uses five quarts. Currently, Taylor has 6,000 quarts of the material in inventory. All of the material is imported. For the coming year, Taylor plans to import 6,000 quarts to produce 1,000 units of Pocolimpio and 2,000 units of Maslimpio. The detail of each product's unit contribution margin is as follows:

Pocolimpio Maslimpio

| Selling price | $\$ 81$ | $\$ 139$ |
| :--- | :--- | :--- |
| Less variable expenses: | $(20)$ | $(50)$ |
| Direct materials | $(21)$ | $(14)$ |
| Direct labor | $\underline{(10)}$ | $\underline{(15)}$ |
| Variable overhead | $\underline{\underline{\$ 30}}$ | $\underline{\underline{\$ 60}}$ |

Taylor Company has received word that the source of the material has been shut down by embargo. Consequently, the company will not be able to import the 6,000 quarts it planned to use in the coming year's production. There is no other source of the material.

## Required:

1. Compute the total contribution margin that the company would earn if it could import the 6,000 quarts of the material.
2. Determine the optimal usage of the company's inventory of 6,000 quarts of the material. Compute the total contribution margin for the product mix that you recommend.
3. Assume that Pocolimpio uses three direct labor hours for every unit produced and that Maslimpio uses two hours. A total of 6,000 direct labor hours is available for the coming year.
a. Formulate the linear programming problem faced by Taylor Company. To do so, you must derive mathematical expressions for the objective function and for the material and labor constraints.
b. Solve the linear programming problem using the graphical approach.
c. Compute the total contribution margin produced by the optimal mix.

## 21-16 Product Mix Decision, Single and Multiple Constraints, Basics of Linear Programming

LO3 Desayuno Products, Inc., produces cornflakes and branflakes. The manufacturing process is highly mechanized; both products are produced by the same machinery by using different settings. For the coming period, 200,000 machine hours are available. Management is trying to decide on the quantities of each product to produce. The following data are available:

## Cornflakes Branflakes

|  | 1.00 | 0.50 |
| :--- | ---: | ---: |
| Machine hours per unit | $\$ 2.50$ | $\$ 3.00$ |
| Unit selling price | $\$ 1.50$ | $\$ 2.25$ |

## Required:

1. Determine the units of each product that should be produced in order to maximize profits.
2. Because of market conditions, the company can sell no more than 150,000 packages of cornflakes and 300,000 boxes of branflakes. Do the following:
a. Formulate the problem as a linear programming problem.
b. Determine the optimal mix using a graph.
c. Compute the maximum contribution margin given the optimal mix.

## 21-17 Product Mix Decisions

LO3 Calen Company manufactures and sells three products in a factory of three departments. Both labor and machine time are applied to the products as they pass through each department. The nature of the machine processing and of the labor skills required in each department is such that neither machines nor labor can be switched from one department to another.

Calen's management is attempting to plan its production schedule for the next several months. The planning is complicated by the fact that labor shortages exist in the community and some machines will be down several months for repairs.

Following is information regarding available machine and labor time by department and the machine hours and direct labor hours required per unit of product. These data should be valid for at least the next six months.

|  | Department |  |  |
| :--- | :--- | :--- | :--- |
| Monthly Capacity | 1 | 2 | 3 |
| Labor hours available | 3,700 | 4,500 | 2,750 |
|  | 3,000 | 3,100 | 2,700 |
| Product | Input per Unit Produced |  |  |
| 401 | Labor hours | 2 | 3 |

Calen believes that the monthly demand for the next six months will be as follows:

| Product | Units Sold |
| :---: | :---: |
| 401 | 500 |
| 402 | 400 |
| 403 | 1,000 |

Inventory levels will not be increased or decreased during the next six months. The unit cost and price data for each product are as follows:

|  | Product |  |  |
| :--- | ---: | ---: | ---: |
|  | 401 | 402 | 403 |
| Unit costs: |  |  |  |
| Direct material | $\$$ | $\$ 13$ | $\$ 17$ |
| Direct labor | 66 | 38 | 51 |
| Variable overhead | 27 | 20 | 25 |
| Fixed overhead | 15 | 10 | 32 |
| Variable selling | $\frac{3}{\$ 118}$ | $\frac{2}{\$ 83}$ | $\underline{4}$ |
| Total unit cost | $\underline{\underline{\$ 196}}$ | $\overline{\underline{\$ 123}}$ | $\overline{\$ 167}$ |

## Required:

1. Calculate the monthly requirement for machine hours and direct labor hours for producing Products 401,402 , and 403 to determine whether or not the factory can meet the monthly sales demand.
2. Determine the quantities of 401,402 , and 403 that should be produced monthly to maximize profits. Prepare a schedule that shows the contribution to profits of your product mix.
3. Assume that the machine hours available in department 3 are 1,500 instead of 2,700 . Calculate the optimal monthly product mix using the graphing approach to linear programming. Prepare a schedule that shows the contribution to profits from this optimal mix. (CMA adapted)

## 21-18 Identifying and Exploiting Constraints, Constraint Elevation

LO4 Berry Company produces two different metal components used in medical equipment (Component X and Component Y ). The company has three processes: molding, grinding, and finishing. In molding, molds are created, and molten metal is poured into the shell. Grinding removes the gates that allowed the molten metal to flow into the mold's cavities. In finishing, rough edges caused by the grinders are removed by small, handheld pneumatic tools. In molding, the setup time is one hour. The other two processes have no setup time required. The demand for Component X is 300 units per day, and the demand for Component Y is 500 units per day. The minutes required per unit for each product are as follows:

|  | Minutes Required per Unit of Product |  |  |
| :--- | :---: | :---: | :---: |
| Product | Molding | Grinding | Finishing |
| Component X | 5 | 10 | 15 |
| Component Y | 10 | 15 | 20 |

The company operates one 8 -hour shift. The molding process employs 12 workers (who each work eight hours). Two hours of their time, however, are used for setups (assuming both products are produced). The grinding process has sufficient equipment and workers to provide 200 grinding hours per shift.

The finishing department is labor intensive and employs 35 workers, who each work eight hours per day. The only significant unit-level variable costs are materials and power. For Component X , the variable cost per unit is $\$ 40$, and for Component Y , it is $\$ 50$. Selling prices for X and Y are $\$ 90$ and $\$ 110$, respectively. Berry's policy is to use two setups per day: an initial setup to produce all that is scheduled for Component X and a second setup (changeover) to produce all that is scheduled for Component Y. The amount scheduled does not necessarily correspond to each product's daily demand.

## Required:

1. Calculate the time (in minutes) needed each day to meet the daily market demand for Component X and Component Y. What is the major internal constraint facing Berry Company?
2. Describe how Berry should exploit its major binding constraint. Specifically, identify the product mix that will maximize daily throughput.
3. Assume that manufacturing engineering has found a way to reduce the molding setup time from one hour to 10 minutes. Explain how this affects the product mix and daily throughput.

## 21-19 Theory of Constraints, Internal Constraints

LO4 Pratt Company produces two replacement parts for a popular line of VCRs: Part A and Part B. Part A is made up of two components, one manufactured internally and one purchased from external suppliers. Part B is made up of three components, one manufactured internally and two purchased from suppliers. The company has two processes: fabrication and assembly. In fabrication, the internally produced components are made. Each component takes 20 minutes to produce. In assembly, it takes 30 minutes to assemble the components for Part A and 40 minutes to assemble the components for Part B. Pratt Company operates one shift per day. Each process employs 100 workers who each work eight hours per day.

Part A earns a unit contribution margin of $\$ 20$, and Part B earns a unit contribution margin of $\$ 24$ (calculated as the difference between revenue and the cost of materials and energy). Pratt can sell all that it produces of either part. There are no other constraints. Pratt can add a second shift of either process. Although a second shift would work eight hours, there is no mandate that it employ the same number of workers. The labor cost per hour for fabrication is $\$ 15$, and the labor cost per hour for assembly is $\$ 12$.

## Required:

1. Identify the constraints facing Pratt, and graph them. How many binding constraints are possible? What is Pratt's optimal product mix? What daily contribution margin is produced by this mix?
2. What is the drummer constraint? How much excess capacity does the other constraint have? Assume that a 1.5 -day buffer inventory is needed to deal with any production interruptions. Describe the drum-buffer-rope concept using the Pratt data to illustrate the process.
3. Explain why the use of local labor efficiency measures will not work in Pratt's TOC environment.
4. Suppose Pratt decides to elevate the binding constraint by adding a second shift of 50 workers (labor rates are the same as those of the first shift). Would elevation of Pratt's binding constraint improve its system performance? Explain with supporting computations.

## 21-20 TOC, Internal and External Constraints

LO4
Bountiful Manufacturing produces two types of bike frames (Frame X and Frame Y). Frame X passes through four processes: cutting, welding, polishing, and painting. Frame Y uses three of the same processes: cutting, welding, and painting. Each of the four processes employs 10 workers who work eight hours each day. Frame X sells for $\$ 40$ per unit, and Frame Y sells for $\$ 55$ per unit. Materials is the only unit-level variable expense. The materials cost for Frame X is $\$ 20$ per unit, and the materials cost for Frame Y is $\$ 25$ per unit. Bountiful's accounting system has provided the following additional information about its operations and products:

| Resource Name | Resource Available | Frame X Resource <br> Usage per Unit | Frame Y Resource <br> Usage per Unit |
| :--- | :---: | :---: | :---: |
| Cutting labor | 4,800 minutes | 15 minutes | 10 minutes |
| Welding labor | 4,800 minutes | 15 minutes | 30 minutes |
| Polishing labor | 4,800 minutes | 15 minutes | - |
| Painting labor | 4,800 minutes | 10 minutes | 15 minutes |
| Market demand: | 200 per day | One unit | - |
| $\quad$ Frame X | 100 per day | - | One unit |

Bountiful's management has determined that any production interruptions can be corrected within two days.

## Required:

1. Assuming that Bountiful can meet daily market demand, compute the potential daily profit. Now, compute the minutes needed for each process to meet the daily market demand. Can Bountiful meet daily market demand? If not, where is the bottleneck? Can you derive an optimal mix without using a graphical solution? If so, explain how.
2. Identify the objective function and the constraints. Then, graph the constraints facing Bountiful. Determine the optimal mix and the maximum daily contribution margin (throughput).
3. Explain how a drum-buffer-rope system would work for Bountiful.
4. Suppose that the engineering department has proposed a process design change that will increase the polishing time for Frame X from 15 to 23 minutes per unit and decrease the welding time from 15 minutes to 10 minutes per unit (for Frame X). The cost of process redesign would be $\$ 10,000$. Evaluate this proposed change. What step in the TOC process does this proposal represent?

## 21-21 Collaborative Learning Exercise

LO1, LO2, The following reasons have been offered for holding inventories:
a. To balance ordering or setup costs and carrying costs
b. To satisfy customer demand (e.g., meet delivery dates)
c. To avoid shutting down manufacturing facilities because of:
(1) Machine failure
(2) Defective parts
(3) Unavailable parts
d. Unreliable production processes
e. To take advantage of discounts
f. To hedge against future price increases

## Required:

Form groups of three to five. Each of the groups will choose one of the letters, "a" through " f ," corresponding to the above reasons for holding inventory. No group can choose a letter chosen by another group until all the letters are used. The letter selection process ends when each group has at least one letter. Each group will determine how the JIT approach responds to their designated reason(s) for holding inventory. The groups will then share their answers with the other groups.

## 21-22 Cyber Research Case

LO4 Please answer each of the following:

1. Go to http://www.goldratt.com, and locate the list of cases detailing successful use of the theory of constraints. Pick three cases, and summarize the benefits each firm realized from implementing TOC.
2. Access the library at http://www.goldratt.com, and see if you can find any information on what TOC followers call the "Thinking Process." If not, then do a general Internet search to find the information. Once located, describe what is meant by the "Thinking Process."

[^0]:    1. Taken from the Web site, http://www.goldratt.com, as of January 19, 2001.
[^1]:    2. $d(T C) / d Q=C / 2-D P / Q^{2}=0$; thus, $Q^{2}=2 D P / C$ and $Q=\sqrt{2 D P / C w}$.
[^2]:    3. Eliyahi M. Goldratt and Robert E. Fox, The Race (Croton-on-Hudson, NY: North River Press, 1986).
[^3]:    4. Jon Bigness, "In Today's Economy There Is Big Money to Be Made in Logistics," The Wall Street Journal (September 6, 1995): A1 and A9.
[^4]:    5. Michael Hammer and James Champy, Reengineering the Corporation (New York: Harper Business, 1993). 6. Gene Schwind, "Man Arrives Just in Time to Save Harley-Davidson," Material Handling Engineering (August 1984): 28-35.
[^5]:    7. Jack Bailes and Ilene K. Kleinsorge, "Cutting Waste with JIT," Management Accounting (May 1992): 28-32. 8. William J. Stoddard and Nolan W. Rhea, "Just-in-Time Manufacturing: The Relentless Pursuit of Productivity," Material Handling Engineering (March 1985): 70-76.
[^6]:    9. Ibid.
[^7]:    * Optimal solution.

[^8]:    10. This follows the definition of Eliyahi Goldratt and Robert Fox in The Race. Other definitions and basic concepts of the theory of constraints are also based upon the developments of Goldratt and Fox.
[^9]:    11. As described in "Success Stories," online at http://www.goldratt.com/success.htm, as of April 7, 1999.
    12. Ibid.
